MATHEMATICS

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AIM POINT
MATHEMATICS
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TRIGONOMETRIC EQUATIONS

& Their Properties

CONTENTS

THINGS TO REMEMBER

✷ Trigonometric Equations :

An equation involving one of more trigonometrical ratios of unknown angle is called a trigonometric equation

eg, $\cos^2\theta - \sin\theta = \frac{1}{2}, \tan m\theta = \cot n\theta$ etc, are trigonometric equations.

✷ Trigonometric Identity :

A relation is said to be a trigonometric identity if it is satisfied by all values of the unknown angle fo which trigonometric ratios involved are defined.

eg, $\sin^2\theta + \cos^2\theta = 1$ and $\sec^2\theta = 1 + \tan^2\theta$ etc, are trigonometric identity, since these are satisfied by all values of θ for which sinθ, cosθ and secθ, tanθ are defined.

If the rotation is in clockwise sense, the angle measured is negative and if the rotation is in anti-clockwise sense, the angle measured is positive.

✷ Solutions or Roots of a Trigonometric Equation :

A value of the unknown angle which satisfies the given equation, is called a solution of root of the equation.

The trigonometric equation may have infinite number of solutions and can be classified as

1. Principal Solution
2. General Solution

1. Principal Solution

The least value of unknown angle which satisfies the given equation, is called a principal solution of trigonometric equation.

2. General Solution

We know that trigonometric function are periodic and solution of trigonometric equations can be generalized with the help of the periodicity of the trigonometric function. The solution consisting of all possible solutions of a trigonometric equation is called its general solution.

General Solution of Trigonometric Equations

1.	$\sin\theta = 0$	$\theta = n\pi, n \in I$
2.	$\cos\theta = 0$	$\theta = (2n+1)\frac{\pi}{2}, n \in I$
3.	$\tan\theta = 0$	$\theta = n\pi, n \in I$
4.	$\sin\theta = \sin\alpha$	$\theta = n\pi + (-1)n\alpha, \alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right], n \in I$
5.	$\cos\theta = \cos\alpha$	$\theta = 2n\pi \pm \alpha, \alpha \in [0, \pi], n \in I$
6.	$\tan\theta = \tan\alpha$	$\theta = n\pi + \alpha, \alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right), n \in I$

7.	$\left.\begin{array}{l}\sin^2\theta = \sin^2\alpha \\ \cos^2\theta = \cos^2\alpha \\ \tan^2\theta = \tan^2\alpha\end{array}\right\}$	$n\pi \pm \alpha, n \in I$
8.	$\sin\theta = 1$	$\theta = (4n\pi + 1)\dfrac{\pi}{2}, n \in I$
9.	$\cos\theta = 1$	$\theta = 2n\pi, n \in I$
10.	$\cos\theta = -1$	$\theta = (2n + 1)\pi, n \in I$

✷ **Solutions of Trigonometric Equation of the Form $a\cos\theta + b\sin\theta = c$:**

Let the equation is

$$a\cos\theta + b\sin\theta = c$$

On dividing by $\sqrt{a^2+b^2}$ both sides, we get

$$\frac{a}{\sqrt{a^2+b^2}}\cos\theta + \frac{b}{\sqrt{a^2+b^2}}\sin\theta = \frac{c}{\sqrt{a^2+b^2}} \qquad ....(i)$$

Let $\quad \tan a = \tan\alpha = \dfrac{b}{a}$

; $\quad \sin\alpha = \dfrac{b}{\sqrt{a^2+b^2}}$

and $\quad \cos\alpha = \dfrac{a}{\sqrt{a^2+b^2}}$

From Eq. (i), we get

$$\cos\theta\cos\alpha + \sin\theta\sin\alpha = \frac{c}{\sqrt{a^2+b^2}}$$

$$\Rightarrow \quad \cos(\theta-\alpha) = \frac{c}{\sqrt{a^2+b^2}}$$

If $\quad |c| > \sqrt{a^2+b^2}$

then equation a cos t + b sin t = c has no solution and if,

$$|c| \le \sqrt{a^2+b^2}$$

then let $\quad \cos\phi = \dfrac{|c|}{\sqrt{a^2+b^2}}$

$$\Rightarrow \quad \cos(\theta - \alpha) = \cos\phi$$

$$\Rightarrow \quad \theta - \alpha = 2n\pi \pm \phi$$

$$\Rightarrow \quad \theta = 2n\pi + \phi + \alpha, n \text{ e } I$$

Some Movement to think !

1. While solving a trigonometric equation, squaring the equation at any step should be avoided as far as pollsible. If squaring is necessary, check the solution for extraneous value.
2. Never cancel terms, containing unknown terms on the two sides, which are in product. It may cause loss of genine solution.
3. The answer sholud not contain such values of angle which make any of the terms undefined of infinte.
4. Domain should not change, if it changes necessary corrections mush be made.
5. Check that denominator is not zero at any stage while solving equations.

Note :

- A trigonometric identity is satisfied by any value of unknown angle while trigonometric equation is satisfied by certain values of unknown angle.
- If θ is an odd multiple of $\frac{\pi}{2}$ ie, when $\theta = (2n+1)\frac{\pi}{2}, n \in I$ then secθ and tanθ are not defined.
- If θ is an even multiple of $\frac{\pi}{2}$ ie, when $\theta = n\pi, n \in I$ then cosecθ and cotθ are not defined.
- The general solution of equations containing cosecθ, secθ and cotθ is equivalent to that of the equation involving sinθ, cosθ, tanθ.
- If value of θ which satisfies the two trigonometric equations, is least positive value α, then $\theta = 2n\pi + \alpha, n \in I$

  eg, $\sin\theta = \sin\alpha$ and $\cos\theta = \cos\alpha$,

  then $\theta = 2n\pi + \alpha, n \in I$

  Where cos t = 0, then sin t = 1 or -1

  $\Rightarrow \quad \theta = \left(n + \frac{1}{2}\right)\pi$

  if sinθ = 1, then n is even and if sinθ = -1, then n is odd.

  Similarly, when sinθ = 0, then cosθ = 1 or -1.

  $\Rightarrow \quad \theta = n\pi$

  If cosθ = 1, then n is even and if cosθ = -1, then n is odd.